Visualizing order parameter spaces and topological defects in ordered media

Univerza v Mariboru
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The 4th Polish-Slovenian International Seminar on Young4SoftMatter DAY
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Soft Matter

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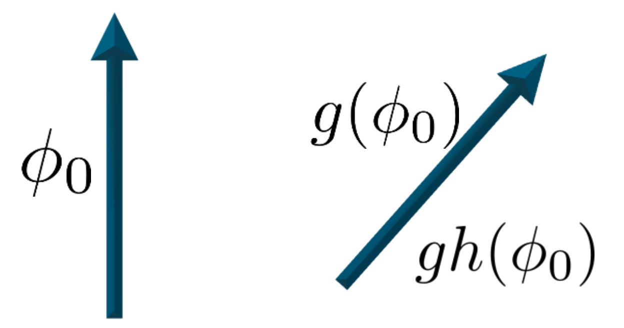
Group theory and symmetries

- Symmetries are described with groups
- Symmetry of the disordered state: G
- Symmetry of *an* ordered state: $H \subset G$

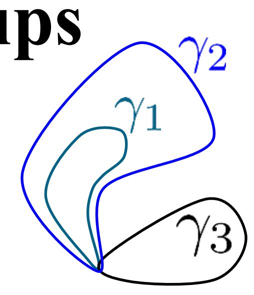


$gH = \{gh \mid h \in H\}$ Cosets represent physically distinguishable ordered states

$G/H = \{gH \mid g \in G\}$ The set of cosets is the order parameter space

Topological defects and fundamental groups

- Line defects are characterized by the first fundamental group of the order parameter space $\pi_1(G/H)$ → Group of homotopic loops in OP space



$\pi_1(G/H) = \tilde{H}/\tilde{H}_0$

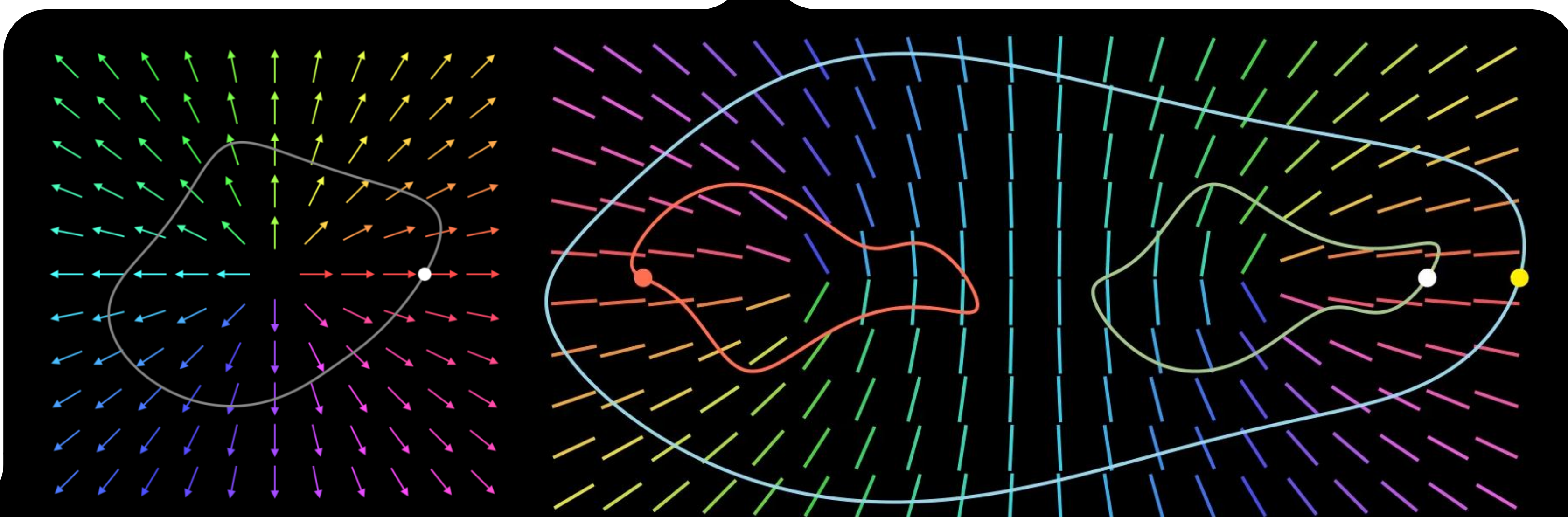
Lift to the simply connected universal covering group

Component containing the identity element

2D Vector order parameter

$G = \mathrm{SO}(2)$ $\quad \mathrm{SO}/e = \mathrm{SO}(2) \cong S^1$

$H = \{e\}$ $\quad \pi_1(S^1) = \mathbb{Z}$



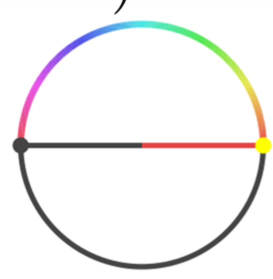
2D Uniaxial order parameter

$G = \mathrm{SO}(2)$

$H = \mathbb{Z}_2 = \{1, -1\}$

$\mathrm{SO}/\mathbb{Z}_2 = \mathbb{RP}^1$

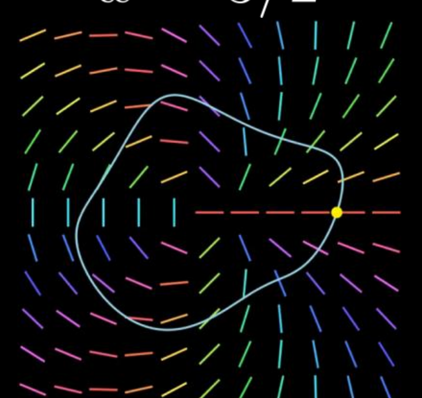
$\pi_1(\mathbb{RP}^1) = \mathbb{Z}$



Defects are labelled by the ***winding number***:

$$w = \frac{1}{2\pi}\oint_\gamma \mathrm{d}\theta$$

$w = 3/2$

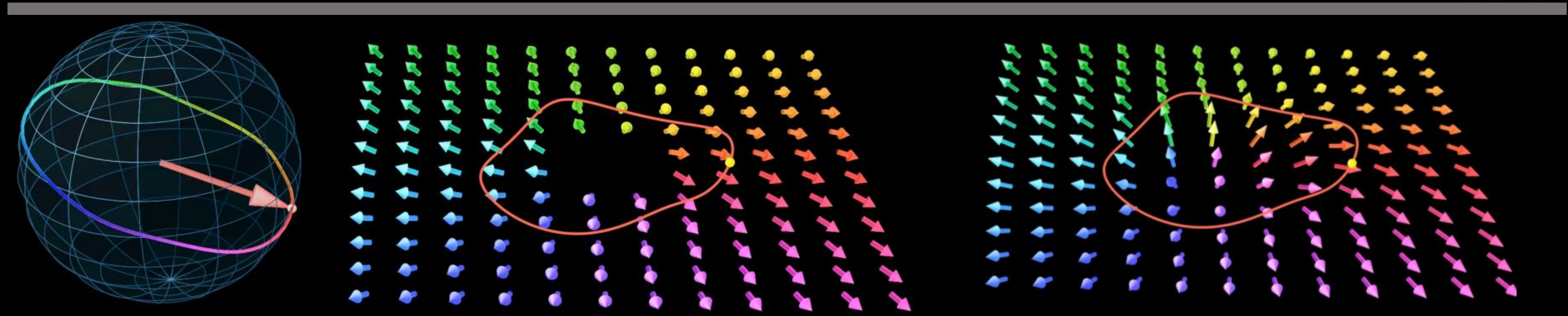


$w = -3/2$

3D Vector order parameter

$G = \mathrm{SO}(3)$ $\quad \mathrm{SO}(3)/\mathrm{SO}(2) = S^2$

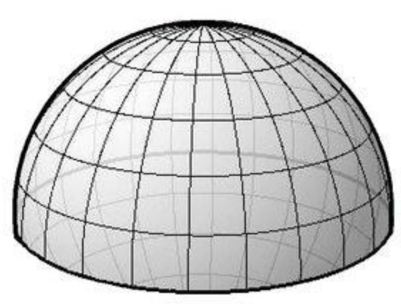
$H = \mathrm{SO}(2)$ $\quad \pi_1(S^2) = \{e\}$



3D Uniaxial order parameter

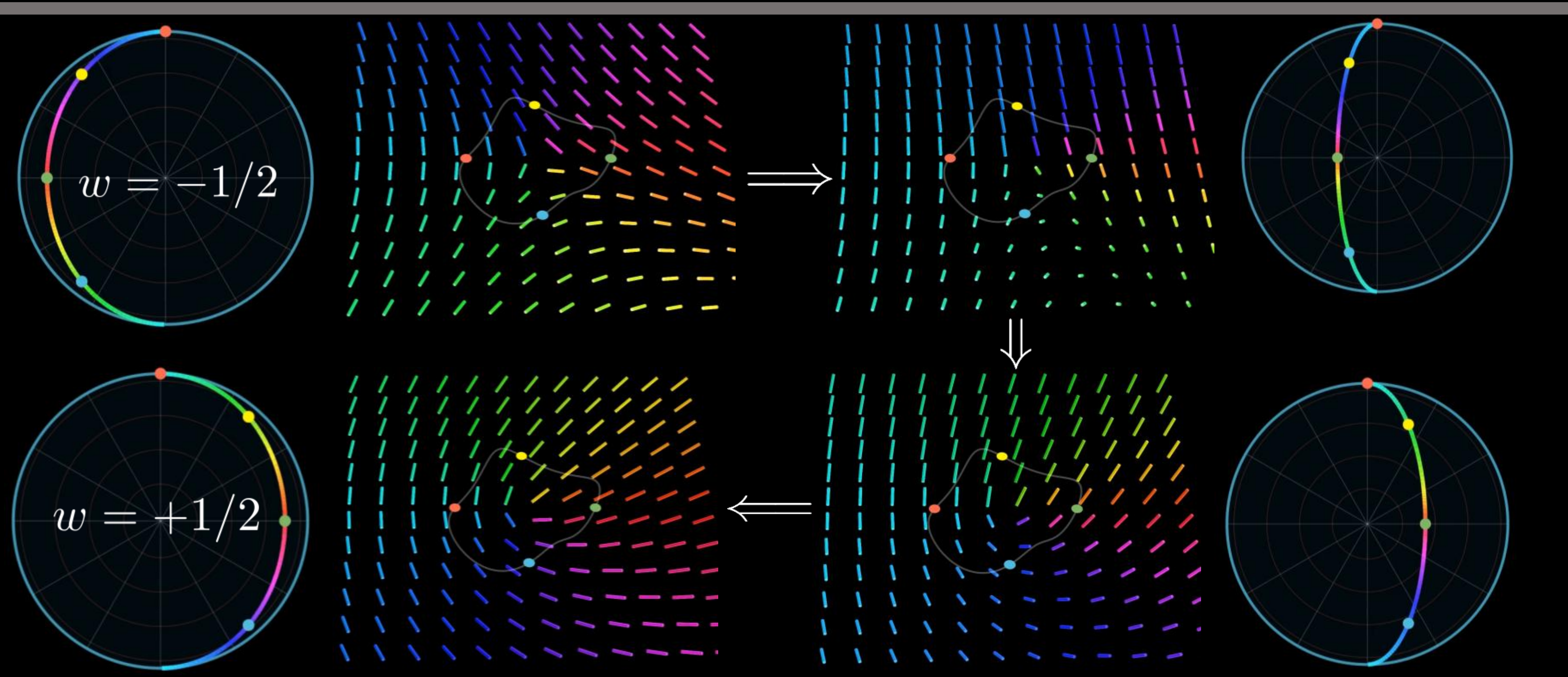
$G = \mathrm{SO}(3)$

$H = D_\infty$



$\mathrm{SO}(3)/D_\infty = \mathbb{RP}^2$

$\pi_1(\mathbb{RP}^2) = \mathbb{Z}_2 = \{1, -1\}$



3D Biaxial order parameter

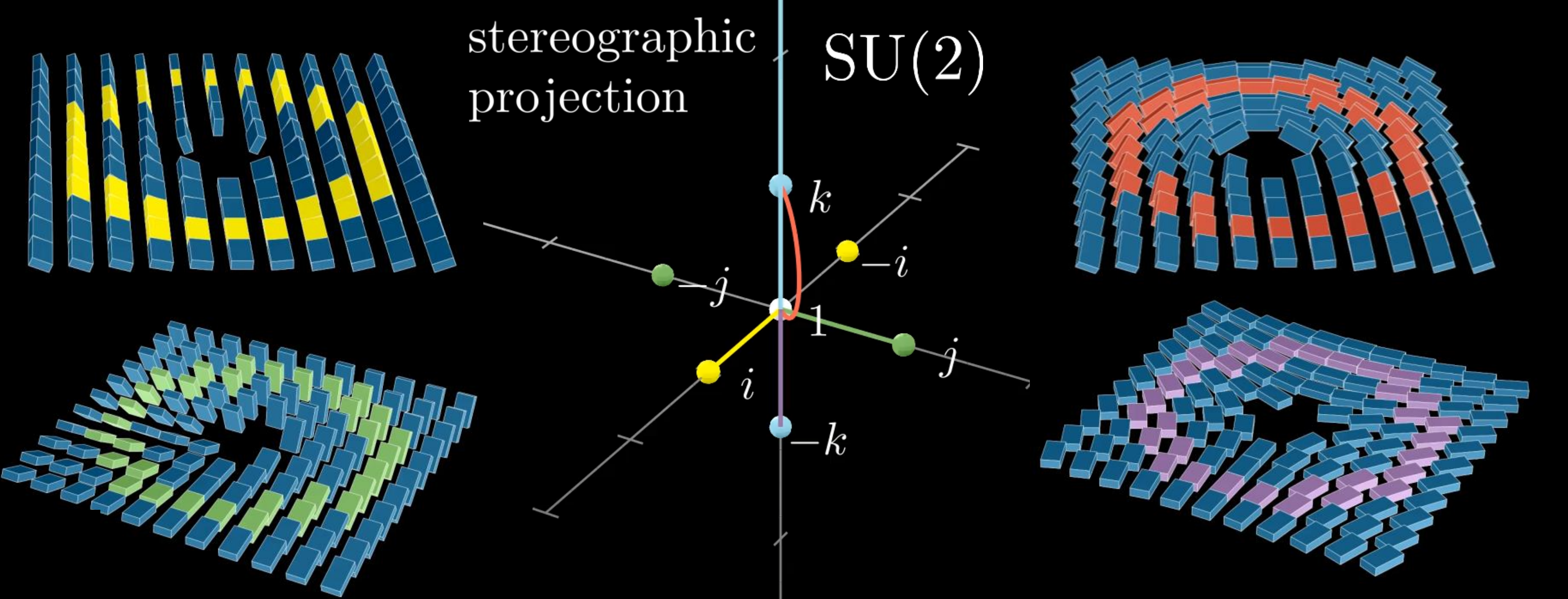
$\pi_1(\mathrm{SO}(3)/D_2) = Q_8$

$Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$

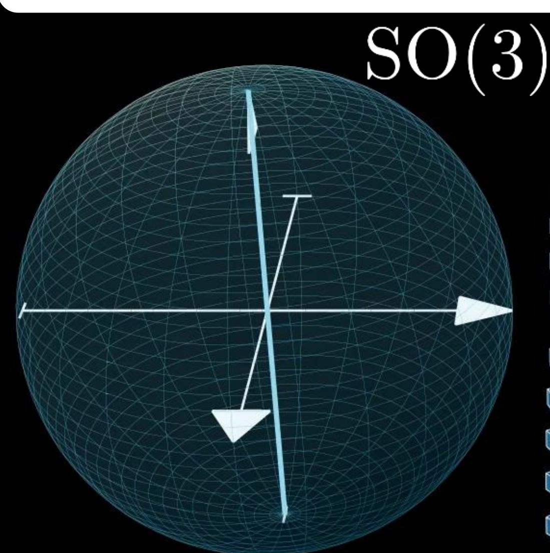
Non-Abelian first fundamental group

$G = \mathrm{SO}(3)$

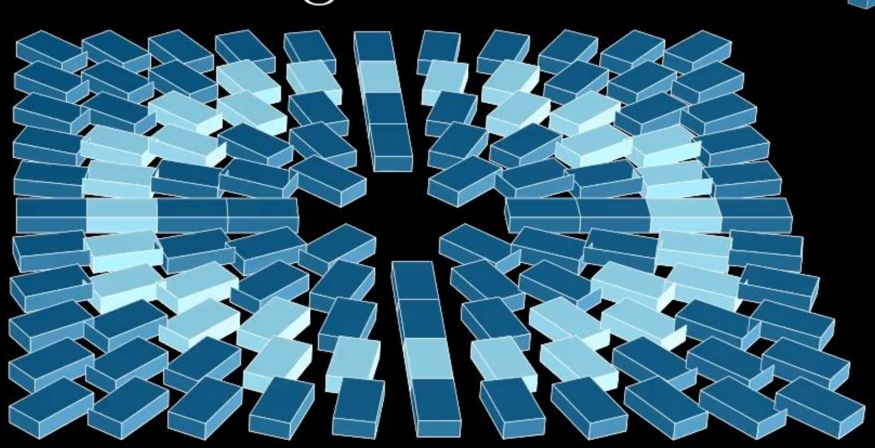
$H = D_2$



SO(3)



"charge" = −1



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