Non-Hermitian Metasurfaces

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Abstract

We describe metasurfaces in terms of real, open systems with non-Hermitian dynamic and effective Hamiltonian. When eigenvalues of the effective Hamiltonian coalesce, we get an Exception Point (EP). We then describe the interaction between light and metasurface with Scattering Matrix and calculate its eigenvalues. Lastly, we show how to connect both with Temporal Coupled Mode Theory and show its applications.

Non-Hermitian Systems

real systems are open

system + environment → density matrix

$$\frac{d\rho}{dt} = -i[H, \rho(t)]$$

von Neumann equation

Hamiltonian $H = H_s + H_E + H_{ES}$ (system, environment, interaction)

$$\frac{d\rho}{dt} = \mathcal{L}\rho \quad \rightarrow \quad \rho(t) = e^{\mathcal{L}t}\rho(0)$$

Liouvillian superoperator → generator of non-Hermitian dynamics

added dissipative processes → jump operator

$$\mathcal{L}(\rho) = -\frac{i}{\hbar}[H, \rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

$$\mathcal{L}(\rho) = -i(H_{eff}\,\rho_s - \rho H_{eff}^\dagger) + \sum L_k \rho L_k^\dagger$$

effective Hamiltonian

$$H_{eff} = H - \frac{i}{2}\sum_k L_k^\dagger L_k$$

→ energy loss

resonators on MS

$$H_{eff} = \begin{pmatrix} \omega_1 - i\gamma_1 & \kappa \\ \kappa & \omega_2 + i\gamma_2 \end{pmatrix}$$

coupling between the resonators

Scattering Matrix

system: scattering channels of light

$$\begin{pmatrix} o_1 \\ o_2 \\ \vdots \\ o_N \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \dots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & S_{NN} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{pmatrix}$$

output, input

reflection only 2D matrix

$$R = \begin{pmatrix} r_0 & r_1 \\ r_2 & r_0 \end{pmatrix}$$

retro reflection, specular reflection

$$\lambda_1 = \lambda_2 = \frac{(S_{11} + S_{22})}{2}$$

poles of the scattering matrix

join MS + Scattering Matrix:

Temporal Coupled Mode Theory

$$S(\omega) = \mathbb{I} - iK^\dagger \frac{1}{\omega\mathbb{I} - H_{eff}} K$$

coupling matrix

$$\det(\omega\mathbb{I} - H_{eff}) = 0$$

poles of S-Matrix = eigenvalues of H_{eff}

Literature

[1] Shou, Y., Wang, D., Wang, Y., *et al.* (2025). Resonant and scattering points in non-Hermitian metasurfaces. *npj Nanophotonics, 2*, 29. https://doi.org/10.1038/s44310-025-00073-6

[2] Bergholtz, E. J., Budich, J. C., & Kunst, F. K. (2021). Exceptional topology of non-Hermitian systems. *Reviews of Modern Physics, 93*(1), 015005. https://doi.org/10.1103/RevModPhys.93.015005

Metasurfaces (MS)

- 2D thin layers
- artificially made materials
- base unit - metaatom - smaller than wavelength of light
- made from combination of dielectrics and metals

Exceptional Point (EP)

eigenvalues of H_{eff}

$$\lambda_\pm = \underbrace{\frac{\omega_1 + \omega_2}{2} - \frac{i(\gamma_1 + \gamma_2)}{2}}_{\lambda_0} \pm \sqrt{\underbrace{\left(\frac{\omega_1 - \omega_2}{2} - \frac{i(\gamma_1 - \gamma_2)}{2}\right)^2}_{\delta} + \kappa^2}$$

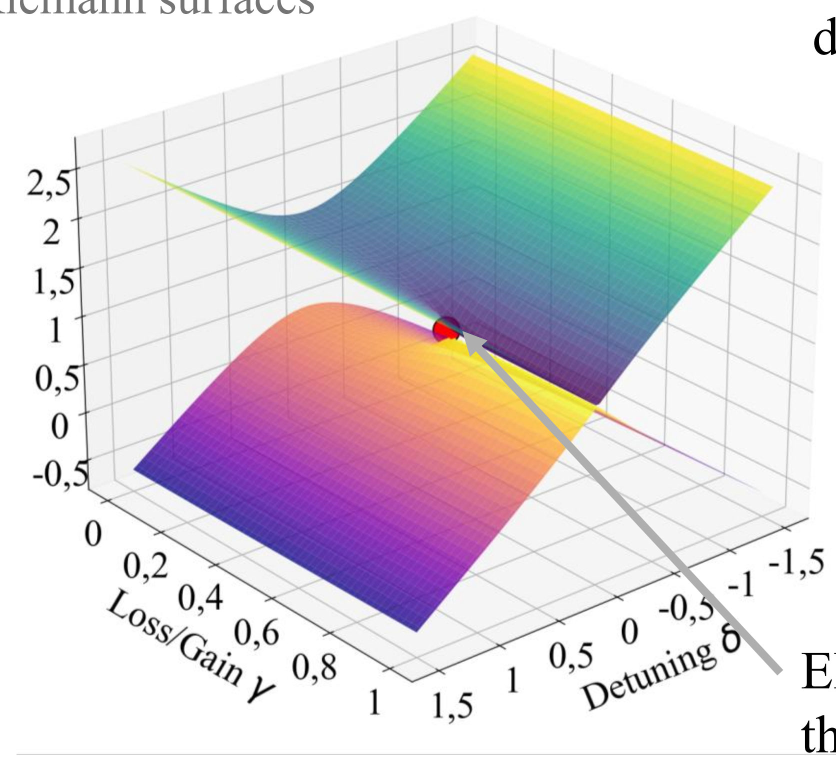
δ → detuning

PT symmetry: $\kappa > \gamma$ — real eigenvalues

broken PT symmetry: $\kappa < \gamma$
- complex eigenvalues
- non-orthogonal eigenstates

$\kappa = \gamma$, $\delta = 0$ → **EP** → topological defects; singularities of non-Hermitian systems

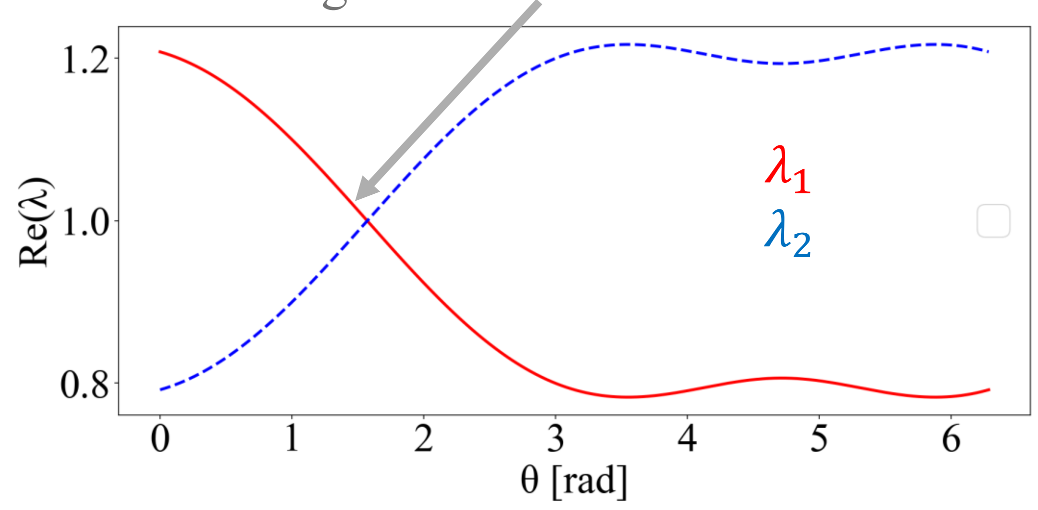
Representation of Eigenvalues with 3D Riemann surfaces



eigenvalues form a complex Riemann surface

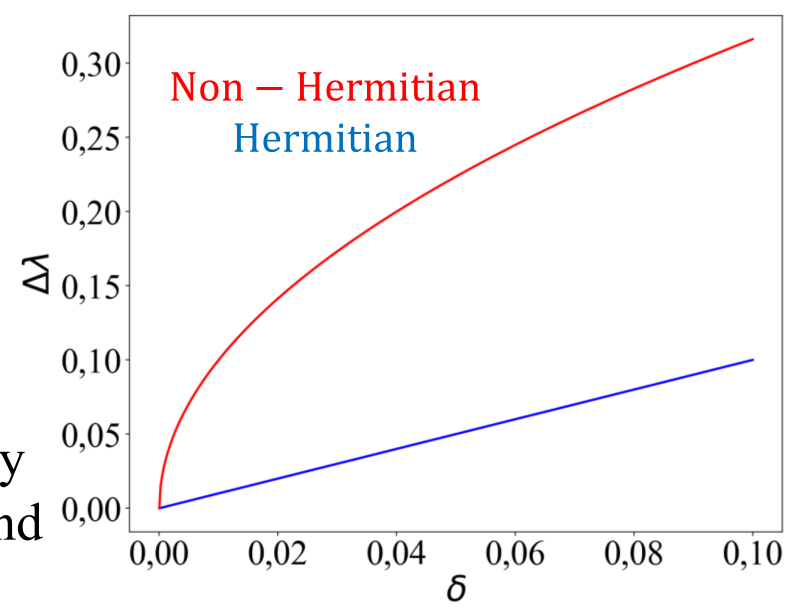
EP - branch point, where they coalesce

Eigenvalues switching around EPs



Applicability

- one-sided invisibility
- perfect absorber
- phase jump
- sensor → enhances sensitivity of the system around EPs



Conclusion

We studied simple two-mode non-Hermitian metasurface and show the emergence of EPs. We connected the effective Hamiltonian with S – Matrix and described the behavior of the system around the EPs.